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K Banhatti and K hyper Banhatti indices of circulant graphs

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ABSTRACT

A topological index is a numerical number associated with a graph that describes its topology. History traces a long path on the study of topological indices. A circulant graph is one of the most comprehensive families, as its specializations give some important families like complete graphs, crown graphs, rook graphs, complete bipartite graphs, cocktail party graphs, empty graphs, etc. The aim of this report is to compute the first and second K Banhatti indices of circulant graph. We also compute the first and second K hyper Banhatti indices of this family of graph. Moreover, we plot our results to see the dependences of the first and second K Banhatti indices and the first and second K hyper Banhatti indices on the involved parameters.

1. Introduction

Chemical graph theory is a branch of graph theory in which a chemical compound is represented by a simple graph called molecular graph in which vertices are atoms of compound and edges are the atomic bonds. A graph is connected if there is at least one connection between its vertices. Throughout this paper, we take G a connected graph. If a graph does not contain any loop or multiple edges then it is called a network. Between two vertices u and v, the distance is the shortest path between them and is denoted by $d(u,v) = d_G(u,v)$ in graph G. For a vertex v of G the “degree” d_v is number of vertices attached with it. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$. The degree and valence in chemistry are closely related to each other. We refer the book (West, 2001) for more details. Nowadays, another emerging field is Cheminformatics, which helps to predict biological activities with the relationship of Structure-property and quantitative structure-activity. Topological indices and Physico-chemical properties are used in prediction of bioactivity if underlined compounds are used in these studies (Rücker and Rücker, 1999; Klavžar and Gutman, 1996).

A number that describe the topology of a graph is called topological index. In 1947, the first and most studied topological index was introduced by Wiener (1947). More details about this index can be found in Dobrynin et al. (2001), Gutman and Polansky (2012), and Randic´ index (Randic, 1975).

Bollobás and Erdös (1998) and Amić et al. (1998), works independently defined the generalized Randic´ index. This index was studied by both mathematicians and chemists (Hu et al., 2005).

The first and second K-Banhatti indices of G are defined as

$$B_1(G) = \sum_{ue \in E(G)}[d_G(u) + d_G(e)]$$

and

$$B_2(G) = \sum_{ue \in E(G)}[d_G(u) \times d_G(e)]$$

where ue means that the vertex u and edge e are incident in G.

The first and second K-hyper Banhatti indices of G are defined as

$$HB_1(G) = \sum_{ue \in E(G)}[d_G(u) + d_G(e)]^2$$

and

$$HB_2(G) = \sum_{ue \in E(G)}[d_G(u) \times d_G(e)]^2$$

We refer (Kulli et al., 2017) for details about these indices.

Let n, m and $a_1, \dots, a_m$ be positive integers, where $1 \le a_i \le \left[\frac{n}{2}\right]$ and $a_i \ne a_j$ for all $i < j < m$. An undirected graph with the set of vertices $V =$

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$\{v_1, \dots, v_n\}$ and the set of edges $E = \{v_i v_{i+a_j}: 1 \le i \le n, 1 \le j \le m\}$, where the indices beign taken modulo n,is called the circulant graph, and is denoted by $C_n(a_1, \dots, a_m)$.

Circulant graph is among the most comprehensive families, as its specializations give some important families. Classes of graphs that are circulant include the, antiprism graphs, crown graphs, cocktail party graphs, rook graphs, complete bipartite graphs, Andrásfai graphs, empty graphs, complete graphs, Paley graphs of prime order, Möbius ladders, torus grid graphs, and prism graphs. Because of this somewhat universality, circulant graphs have been the subject of much investigation; for example, the chromatic index, Connectivity, Wiener index, domination number, revised Szeged spectrum, Multi-level and antipodal labeling, M polynomial and many degree-based topological indices for circulant graphs are studied (Voigt and Walther, 1991; Boesch, and Tindell, 1984; Zhou, 2014; Xueliang et al., 2011; Habibi and Ashrafi, 2014; Kang et al., 2016; Nazeer et al., 2015; Munir et al., 2016). For details on topological indices readers are refered to Sardar et al. (2017) and Rehman et al. (2017).

In this paper, we compute the first and second K Banhatti indices of Circulant Graphs. Moreover we ploted our results (Figs. 2- 5) to see the dependence of our results on the involved structural parameters.

2. Main results

In this section we give our computational results.

Theorem 1. Let G $G = C_n(a_1, a_2, \dots, a_n)$ be Circulant graph. Then the first and the second K Banhatti indices are $n(6n - 10)$ and $n(4n^2 - 12n + 8)$.

Proof. Let $C_n(a_1, a_2 \dots, a_m)$ where $n = 3,4, \dots n$ and $1 \le a_i \le \left[\frac{n}{2}\right]$ and $a_i \ne a_j$ when n is even and when $1 \le a_i \le \left[\frac{n}{2}\right]$ $a_i < a_j$ when n is odd be the circulant graph.From the structure of $C_n(a_1, a_2, \dots, a_m)$,we can see that there is one partition $V_{\{1\}} = \{v \epsilon V(C_n(a_1, a_2, \dots, a_m)) | d_v = n\}$. It is obvo=ious from Fig. 1 that the edge set $C_n(a_1, a_2, \dots, a_m)$ partitions as follow:

$E_{\{n-1,n-1\}} = \{e = uv \epsilon E(C_n(a_1, a_2, \dots, a_m)) | d_u = n - 1 \; and \; d_v = n - 1\} \rightarrow |E_{\{n-1,n-1\}}| = n$

Details of vertices and edges set are given in Table 1.

By definition, we have

$B_1(G) = \sum_{uv \epsilon E(G)} [d_G(u) + d_G(e)]$
$= \sum_{E_{(n-1,n-1)}} [\{d_G(u) + d_G(e)\} + \{d_G(u) + d_G(e)\}]$
$= n[(n - 1 + 2n - 4) + (n - 1 + 2n - 4)]$
$= n(6n - 10)$
$B_2(G) = \sum_{uv \epsilon E(G)} [d_G(u) \times d_G(e)]$
$= \sum_{E_{(n-1,n-1)}} [\{d_G(u) \times d_G(e)\} + \{d_G(u) \times d_G(e)\}]$
$= n[(n - 1) \times (2n - 4) + (n - 1) \times (2n - 4)]$
$= n[(2n^2 - 6n + 4) + (2n^2 - 6n + 4)]$
$= (4n^2 - 12n + 8)$

Table 1: Details of vertices and edges set

(d_u, d_v)	*(n-1,n-1)*
Number of edges	*n*
$(d_G(G)$	*n-1+n-1-2=2n-4*

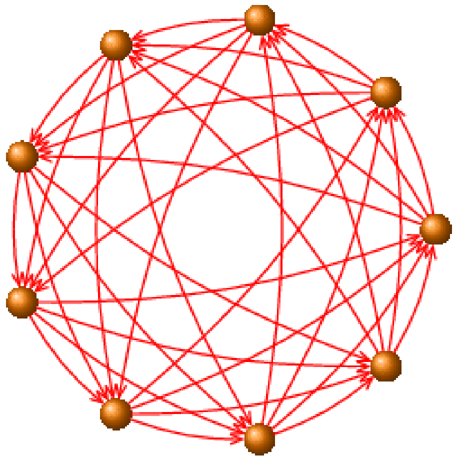

Fig. 1: A circulant graph

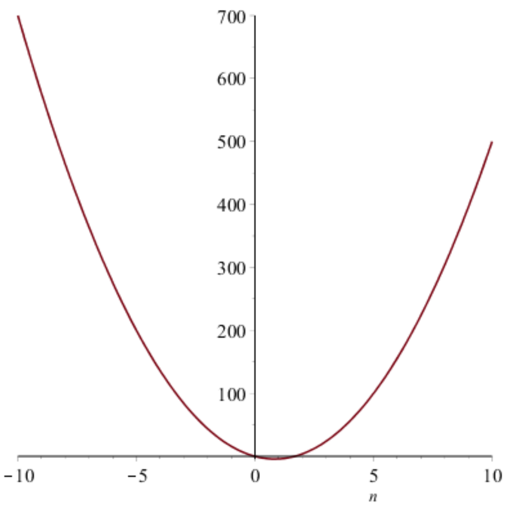

Fig. 2: Plot of first K Banhatti Index

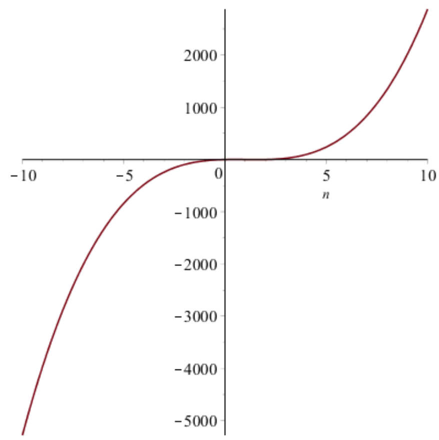

Fig. 3: Plot of second K Banhatti Index

Theorem 2. Let $G = C_n(a_1, a_2, \dots, a_n)$ be the circulant graph. Then the first and second K Hyper Banhatti indices are $2n(3n - 5)^2$ and $2n(2n^2 - 5n + 4)$.

Proof.

$HB_1(G) = \sum_{uv\epsilon E(G)}[d_G(u) + d_G(e)]^2$
$= n[\{(n-1) + (2n-4)^2\} + \{(n-1) + (2n-4)^2\}]$
$= n[2\{(n-1) + (2n-4)^2\}]$
$= 2n(3n-5)^2$

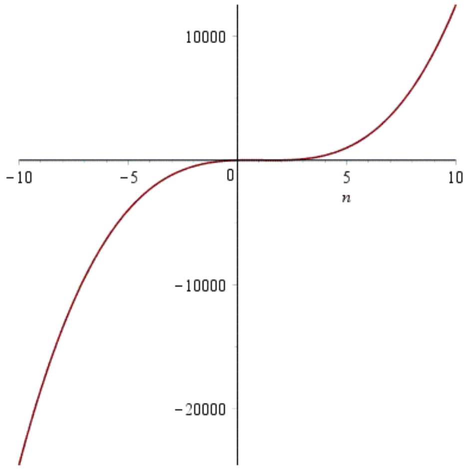

Fig. 4: Plot of first K-hyper Banhatti Index

$HB_2(G) = \sum_{uv\epsilon E(G)}[d_G(u) \times d_G(e)]^2$
$= n[\{(n-1) \times (2n-4)^2\} + \{(n-1) \times (2n-4)^2\}]$
$= n[2\{(n-1) \times (2n-4)^2\}]$
$= 2n((2n^2 - 5n + 4))$

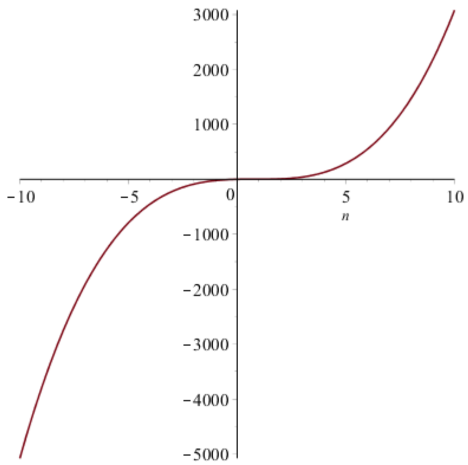

Fig. 5: Plot of second K-hyper Banhatti Index

3. Conclusion

In this article, we computed the first and second K Banhatti indices and the first and second K hyper Banhatti indices of circulant. We plot our results in Figs. 2-5. Our results can play a vital role in pharmacy.

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